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TRANSIENT AXIAL RESPONSE OF A GUN LAUNCHED ROCKET MOTOR CASE DURING LAUNCH

David Salinas, et al

Naval Postgraduate School
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29 January 1973

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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



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by

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and

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NAVAL POSTGRADUATE SCHOOL
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Rear Admiral Mason Freeman
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ABSTRACT:

An analysis for the transient axial response of a gun launched motor case with unbonded propellant is performed. The present effort extends a previous work by including higher harmonics in the representation of the breech force at the aft end of the motor case. Including the higher harmonics leads to a more accurate representation of the actual breech force. The results of the analysis indicate that the dynamic effects on the internal axial force in the case are negligible for a breech force duration of 0.026 seconds. This is the same conclusion that was made in the previous work based upon a simple representation of the breech force.

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# NOTATION

| | |
|---|---|
| $A$ | area of motor case cross section |
| $A_k$ | Fourier cosine coefficients of breech force |
| $B_k$ | Fourier sine coefficients of breech force |
| $c$ | speed of wave propagation |
| $e$ | axial strain |
| $E$ | Young's modulus of elasticity |
| $f$ | axial force |
| $K$ | constant |
| $\ell$ | length |
| $m$ | mass of motor case |
| $P$ | breech force |
| $t$ | time |
| $u$ | deformation |
| $\bar{u}$ | displacement |
| $u$ | rigid body displacement |
| $U$ | transformation variable |
| $v$ | transformation variable |
| $x$, $y$ | coordinate axes |
| $X$, $Y$ | coordinate axes |
| $\alpha$ | time duration of breech force |
| $\gamma$ | constant |
| $\rho$ | mass density of the motor case material |
| $\sigma$ | axial stress |
| $\omega$ | constant |

## INTRODUCTION

This report examines the transient axial response of a gun-launched rocket motor, with unbonded propellant, during the launch phase. It is part of a continuing effort to develop structural design procedures for gun-launched rockets. In a previous report (Ref. 1), an analysis for the time-dependent axial force distribution in the motor case was performed in which the force at the breech end of the case was approximated by a one-half sine wave over the duration of the loading. That analysis established that for the one-half sine wave input, a time duration of .012 seconds leads to negligible dynamic effects, i.e., for a one half sine wave of .012 seconds duration, the internal axial force at any location along the case appears as a duplicate of the breech force, but of smaller magnitude. It was shown, however, that decreasing the time of the force duration to .0012 seconds leads to significant dynamic effects. This result indicated that higher harmonics in the breech force could cause important dynamic effects. Since the actual breech force is not a simple half sine wave, but in fact is a complicated function of time, an investigation was made to consider the problem further.

To account for the higher harmonics in the breech-force, the analysis decomposes the actual force into a Fourier series with 27 terms. This force is applied to the case, and the internal axial force is computed at several locations along the case over the duration of the loading. This analysis then establishes the contribution of the higher harmonics in the breech force to the dynamic stresses in the bar. The analysis is restricted to the free-free motor case.

## ANALYSIS

In the figure below, AB is the motor case, treated as a one-dimensional bar, under the applied breech force P(t). The actual displacement of any point of the bar with respect to the X, Y, fixed coordinate system is denoted $\bar{u}$. This motion consists of two parts, a rigid body motion $\tilde{u}$ and a strain-causing deformation u.


Then

$$\bar{u} = \tilde{u} + u \qquad (1)$$

If m is the total mass of the bar, then the rigid body motion is

$$\frac{d^2\tilde{u}}{dt^2} = \frac{P(t)}{m} \qquad (2)$$

The differential equation of motion for any point of the bar is

$$\frac{\partial^2\bar{u}}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0 \qquad (3)$$

where c is the speed of wave propagation, and x is a coordinate axis parallel to x moving with velocity $\dot{u}$. The constant c is given by

$$c = \sqrt{\frac{E}{\rho}} \qquad (4)$$

where E is Young's modulus of elasticity of the bar material and $\rho$ its specific gravity. Combining equations (1) and (3) gives

$$\frac{\partial^2}{\partial t^2} (\tilde{u} + u) - c^2 \frac{\partial^2 u}{\partial x^2} = 0$$

Substituting for $\frac{\partial^2 \bar{u}}{\partial t^2}$ from (2) yields,

$$\frac{\partial^2 u}{\partial t^2} + \frac{P(t)}{m} - c^2 \frac{\partial^2 u}{\partial x^2} = 0.$$

With the notation $(\ )_{xx}$ denoting $\frac{\partial^2 (\ )}{\partial x^2}$, and $(\ )_{tt}$ denoting $\frac{\partial^2 (\ )}{\partial t^2}$, the governing differential equation is

$$u_{tt} - c^2 u_{xx} = - \frac{P(t)}{m} \qquad \begin{array}{l} t > 0, \\ 0 < x < \ell \end{array} \tag{5}$$

The initial conditions and boundary conditions for the free-free system are

$$\text{I.C.'s:} \quad u(x,o) = o \qquad u_t(x,o) = o \tag{6}$$

$$\text{B.C.'s:} \quad u_x(o,t) = \frac{P(t)}{AE} \quad u_x(\ell,t) = o \tag{7}$$

The solution of equations (5) through (7) gives the deformation of any point for any time greater than zero. The stress is then simply obtained from Hooke's Law, $\sigma = Eu_x$.

The solution method proceeds by transformation of the given system to a system with homogeneous boundary conditions. This is achieved by considering a solution in the form

$$u(x,t) = v(x,t) + U(x,t). \tag{8}$$

Boundary condition equations (7) become

$$\begin{aligned} v_x(o,t) &= \frac{P(t)}{EA} - U_x(o,t) \\ v_x(\ell,t) &= -U_x(\ell,t) \end{aligned} \tag{9}$$

These equations become homogeneous when $U(x,t)$ is taken as

$$U(x,t) = (x - \frac{x^2}{2\ell}) \frac{P(t)}{AE} \tag{10}$$

With equations (8) and (10), governing equations (5) through (7) become

$$v_{tt} - c^2 v_{xx} = f(x,t) \tag{11}$$

$$\text{I.C.'s:}\quad v(x,o) = -U(x,o) \qquad v_t(x,o) = -U_t(x,o) \tag{12}$$

$$\text{B.C.'s:}\quad v_x(o,t) = o \qquad v_t(\ell,t) = o \tag{13}$$

where

$$\begin{aligned} f(x,t) &= -U_{tt} + c^2 U_{xx} - \frac{P(t)}{\rho \ell A} \\ &= -\left(x - \frac{x^2}{2\ell}\right) \frac{P_{tt}}{AE} - \frac{2P}{\rho \ell A} \end{aligned} \tag{14}$$

A Fourier series solution which satisfies boundary conditions (13) is assumed for $v(x, t)$, i.e.,

$$v(x,t) = \sum_{n=0}^{\infty} v_n(t) \cos \frac{n\pi x}{\ell} \tag{15}$$

Accordingly, equations (11) and (12) become

$$\frac{d^2 v_n(t)}{dt^2} + \omega_n^2 v_n(t) = f_n(t) \qquad n = 1, 2, \ldots \tag{16}$$

$$\text{I.C.'s:}\quad v_n(o) = o$$

$$\frac{dv_n}{dt}(o) = \frac{2}{\ell} \int_0^{\ell} \frac{\left(x - \frac{x^2}{2\ell}\right)}{EA} \frac{dP(t)}{dt} \cos \frac{n\pi x}{\ell}\, dx \tag{17}$$

where

$$f_n(t) = \frac{2}{\ell} \int_0^{\ell} f(x,t) \cos \frac{n\pi x}{\ell}\, dx, \tag{18}$$

and

$$\omega_n = \frac{n\pi c}{\ell} \tag{19}$$

To this point the analysis is identical to that presented in reference 1. In that analysis, $P(t)$ was approximated as a half sine wave $P_o \sin \pi t/\alpha$; here, the breech force is approximated by the Fourier expansion,

$$P(t) = A_o + \sum_{k=1}^{N} \left\{ A_k \cos \frac{2k\pi t}{\alpha} + B_k \sin \frac{2k\pi t}{\alpha} \right\} \tag{20}$$

For the linear system, the response due to equation (20) is obtained by superposition of individual terms.

(i) for the uniform force $A_o$:

From equation (14), $f(x,t) = - \frac{2P}{\rho \ell A}$. Then equation (18) gives $f_n = o$ for all $n$. The initial conditions are $v_n = o$, and $\dot{v}_n = o$ from equation (17), and hence $v_n(t) = o$. Then the stress is

$$\sigma = Eu_x = EU_x = (1 - \frac{x}{\ell}) \frac{A_o}{A} \tag{21}$$

This is the stress due to the "rigid body" acceleration for constant force.

(ii) for the cosine terms, $A_k \cos \frac{2k\pi t}{\alpha}$ :

From equation (14) we obtain,

$$f(x,t) = \left\{ \frac{\left(x - \frac{x^2}{2\ell}\right)}{EA} \cdot \left(\frac{2k\pi}{\alpha}\right)^2 - \frac{2c^2}{EA\ell} \right\} \cdot A_k \cos \frac{2k\pi t}{\alpha}$$

Then equations (17) and (18) give

$$\frac{d}{dt} v_n(o) = o \tag{22}$$

$$f_n(t) = - \frac{8k^2\ell}{\alpha^2 n^2 EA} A_k \cos \frac{2k\pi t}{\alpha} \tag{23}$$

With equations (22) and (23), the solution of equation (16) is

$$v_n(t) = \frac{K_n \alpha^2 A_k}{\pi^2(\gamma_n^2 - 1)} \left\{ \cos \frac{2k\pi t}{\alpha} - \cos \omega_n t \right\} \quad (24)$$

where

$$K_n = - \frac{2\ell}{n^2 \alpha^2 EA} \quad (25)$$

and

$$\gamma_n = \frac{\omega_n}{\frac{2k\pi}{\alpha}} \cdot \quad (26)$$

Then the stress associated with $A_k \cos \frac{2k\pi t}{\alpha}$ terms is

$$\sigma = Eu_x = E(v_x + U_x)$$

$$= \sum_{n=1}^{\infty} \frac{2}{\pi n A} \cdot \frac{A_k}{(\gamma_n^2 - 1)} \left\{ \cos \frac{2k\pi t}{\alpha} - \cos \omega_n t \right\} \cdot \sin \frac{n\pi x}{\ell}$$

$$+ \left( 1 - \frac{x}{\ell} \right) \frac{A_k}{A} \cos \frac{2k\pi t}{\alpha} \quad (27)$$

The latter term in equation (27) is the contribution associated with the "rigid body" acceleration.

iii) for sine terms, $B_k \sin \frac{2k\pi t}{\alpha}$ :

Equation (14) gives

$$f(x,t) = \left\{ \frac{x - \frac{x^2}{2\ell}}{EA} \left( \frac{2k\pi^2}{\alpha} \right)^2 - \frac{2c^2}{EA\ell} \right\} A_k \cos \frac{2k\pi t}{\alpha}$$

From equations (17) and (18),

$$\frac{d}{dt} v_n(o) = - \frac{4k\ell B_k}{EAn^2 \pi\alpha} \quad (28)$$

$$f_n(t) = \frac{8k^2\ell}{\alpha^2 n^2 EA} B_k \sin \frac{2k\pi t}{\alpha} \quad (29)$$

The solution of equation (16) with equations (28) and (29) is

$$v_n(t) = \frac{2K_n k B_k \alpha^2}{R_n \pi^2} \left\{ \frac{\gamma_n^2 - 2}{\gamma_n^2 - 1} \right\} \cdot \sin \omega_n t + \frac{K_n \alpha^2 B_k}{\pi^2(\gamma_n^2 - 1)} \sin \frac{2k\pi t}{\alpha} \quad (30)$$

where

$$R_n = \frac{\omega_n}{(\pi/\alpha)} \quad (31)$$

The stress associated with $B_k \sin \frac{2k\pi t}{\alpha}$ is

$$\sigma = \sum_{n=1}^{\infty} \frac{2B_k}{n\pi A} \left\{ \frac{2k}{R_n} \left( \frac{\gamma_n^2 - 2}{\gamma_n^2 - 1} \right) \sin \omega_n t + \frac{1}{\gamma_n^2 - 1} \sin \frac{2k\pi t}{\alpha} \right\} \sin \frac{n\pi x}{\ell}$$

$$+ \left(1 - \frac{x}{\ell}\right) \frac{B_k}{A} \sin \frac{2k\pi t}{\alpha} \quad (32)$$

Again the latter term in equation (32) results from the "rigid body" acceleration.

The stress at any point is obtained as the superposition of equations (21), (27), and (32).

## RESULTS AND DISCUSSION

Equations (21), (27), and (32) for the stress have been evaluated on the NPS - IBM 360/67 digital computer for specific values of $A_k$ and $B_k$ obtained by a Fourier decomposition of the given breech force. This decomposition is done by the IBM subroutine FORIT on the computer. The present analysis resolved the breech force into 26 sine and cosine harmonics, i.e.,

$$P(t) = A_o + \sum_{k=1}^{26} \left\{ A_k \cos \frac{2k\pi t}{\alpha} + B_k \sin \frac{2k\pi t}{\alpha} \right\} \tag{33}$$

The magnitude of the harmonic coefficients $A_k$ and $B_k$ depend upon the nature of the breech force. For this analysis, the force given in Fig. 1 of Ref. 2 was used. Approximation of that force by equation (33) leads to

| k | $A_k$ | $B_k$ |
|---|---|---|
| 0 | 14.1680 | 0. |
| 1 | -8.6082 | 9.6038 |
| 2 | -3.2336 | -4.3103 |
| 3 | 1.3139 | -.8107 |
| 4 | -.1162 | .1077 |
| 5 | .1089 | -.1257 |
| 6 | -.2036 | .3394 |
| 7 | -.3478 | -.0886 |
| 8 | -.3001 | -.1664 |
| 9 | -.2229 | -.3071 |
| 10 | .1728 | -.2293 |
| 11 | .2260 | .1067 |
| 12 | -.0425 | .3243 |
| 13 | -.3157 | .2287 |

| k | $A_k$ | $B_k$ |
|---|---|---|
| 14 | -.3164 | -.0186 |
| 15 | -.1682 | -.0940 |
| 16 | -.0552 | .0146 |
| 17 | -.1202 | .0983 |
| 18 | -.2053 | .0899 |
| 19 | -.1821 | .0827 |
| 20 | -.2162 | .0815 |
| 21 | -.2723 | .0286 |
| 22 | -.2164 | -.0294 |
| 23 | -.1599 | .0250 |
| 24 | -.2146 | .0638 |
| 25 | -.2589 | -.0065 |
| 26 | -.2162 | .0034 |

A plot of equation (33) with the above values of $A_k$ and $B_k$ is given in Fig. 1 for a .026 second time duration of breech force. Figures 2 through 5 are the results for the stress in the bar due to the breech force shown in Fig. 1 as a function of time for $\frac{x}{\ell}$ ratios of 0, 1/4, 1/2, and 3/4, respectively. (The analysis is for the 5 in. steel shell of 0.19 in. thickness). Comparing Fig. 1 with Figs. 2 through 5 reveals that the internal force appears as a duplicate of the breech force, with the stress curves scaled versions of the force input. In Figs. 6 through 8, the contributing harmonics are individually displayed. The elastic dynamic effects are the small oscillations about each harmonic itself.

Figures 10 through 14 are the results of the analysis for a time duration of 0.0052 seconds for the brech force. This is a reduction in the actual duration by a factor of five. This sample problem was considered in order to assess the influence of the magnitude of the force duration. The elastic dynamic effects can be clearly seen in these figures. Note that the magnitude of stress has been significantly changed by the dynamic effects.

## CONCLUSIONS

The results of this analysis (Figs. 1-5) show that dynamic effects are negligible for the breech force of 0.026 seconds time duration of Figure 1, Reference 2. Hence, when the propellant is not bonded to the motor case, the case responds to the breech force as if it were applied in a static sense, i.e., the maximum internal axial load in the case can be computed on the basis of the mass

distribution and the maximum rigid body acceleration of the rocket. This is the same conclusion arrived at in Ref. 1 based on a simple one-half sine wave pulse; hence, accounting for the more complicated actual breech pressure pulse confirmed the original conclusion.

Further investigation (Figs. 10-14) verified the dependence of the dynamic effects on the presence of significant higher harmonic terms in the breech force. Such harmonics may be the result of shortening the time duration of loading, or by the presence of a local rapid perturbation of force on an otherwise acceptable smooth breech force input.

## REFERENCES


1. Ball, Robert E., and Salinas, David, "Analysis of a Three Inch Gun Launched Finned Motor Case," Naval Postgraduate School, NPS-57Bp-72011A, January 1972.

2. Ball, Robert E., "Survivability of the Five-Inch Gun Launched Finned Motor Case," Naval Postgraduate School, NPS-57Bp72081A, August 1972.

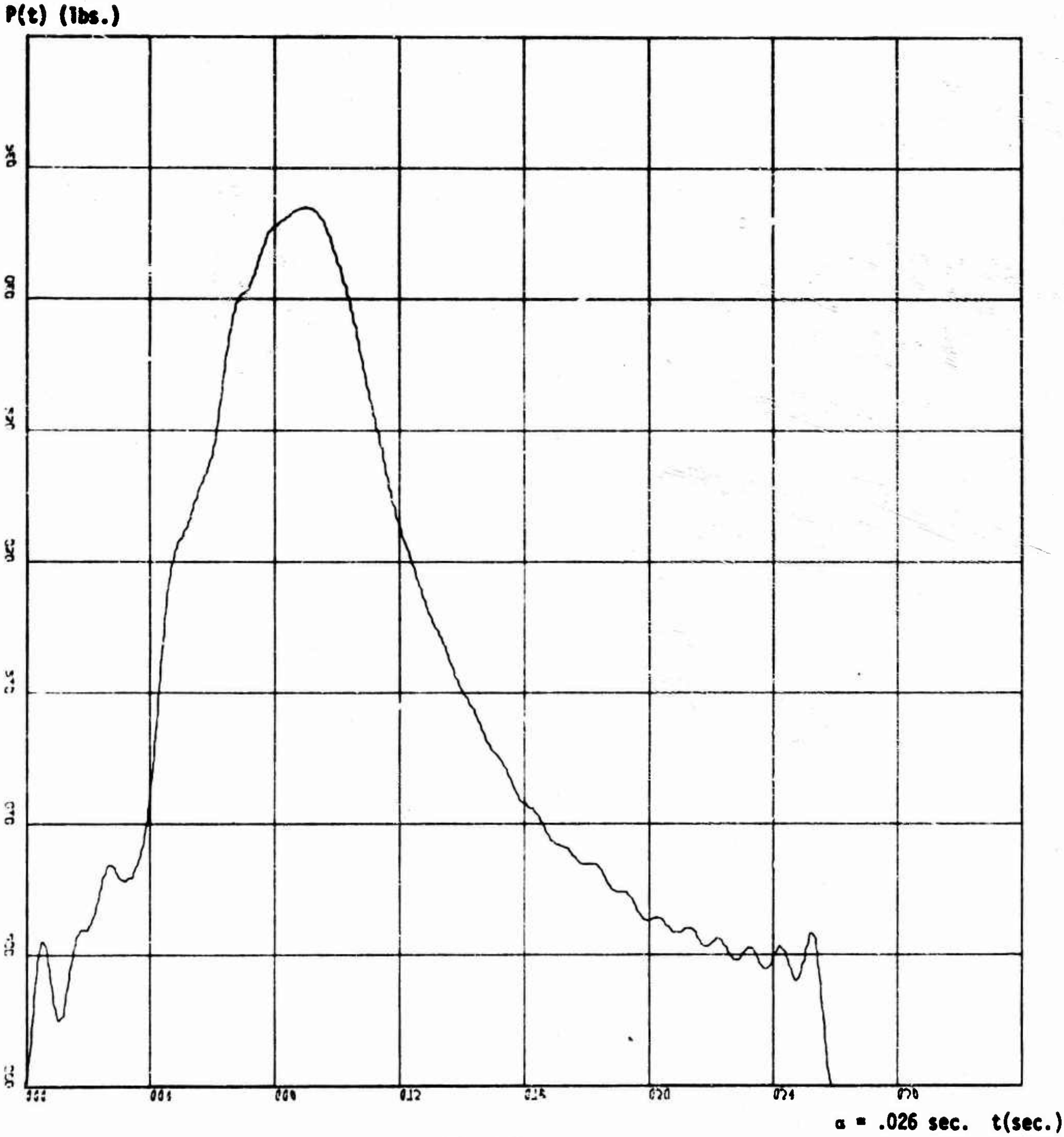



Figure 1

X-SCALE=4.00E-03 UNITS
Y-SCALE=5.00E+00 UNITS
AXIAL WAVE PROPAGATION-BREECH FORCE

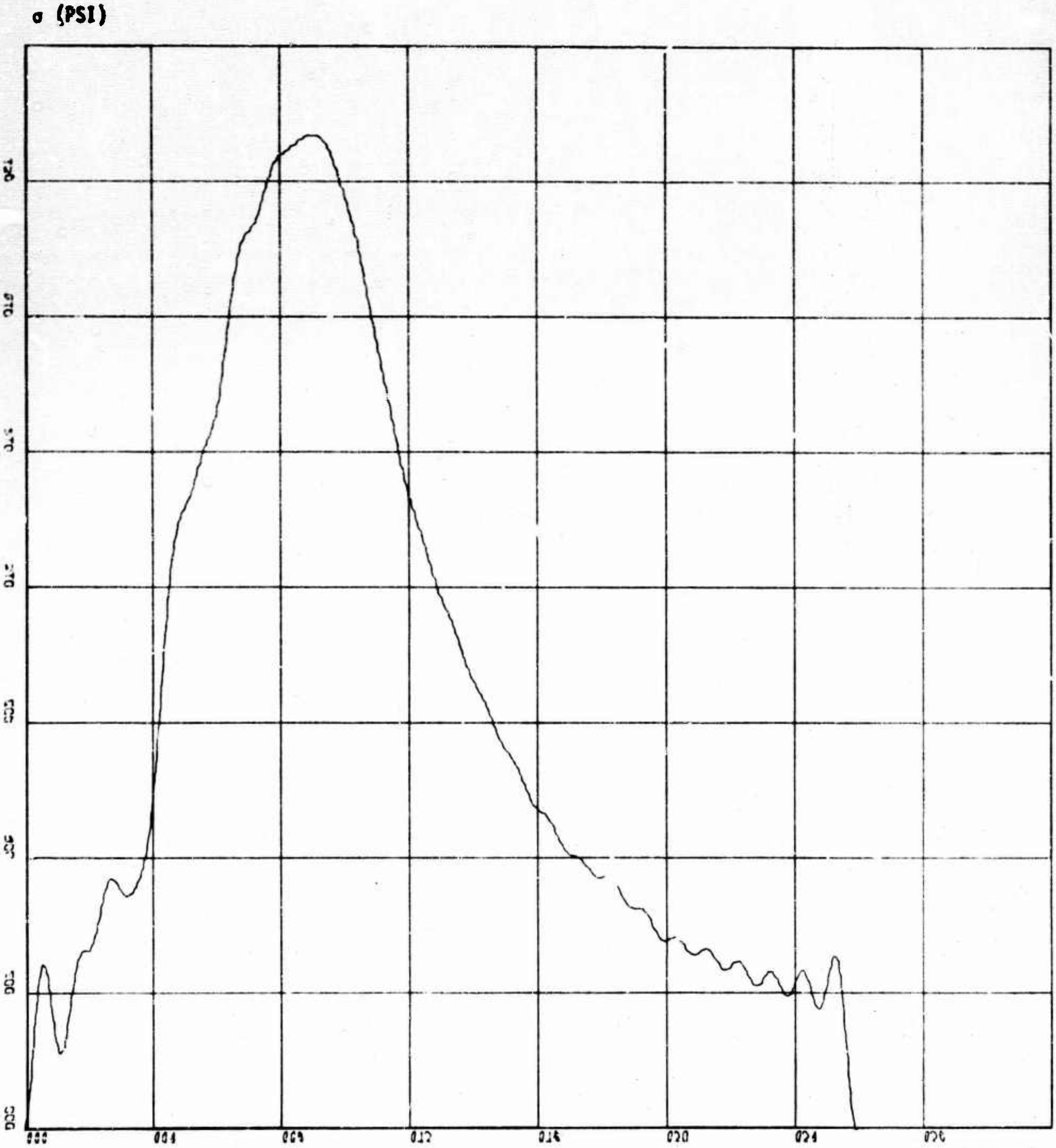
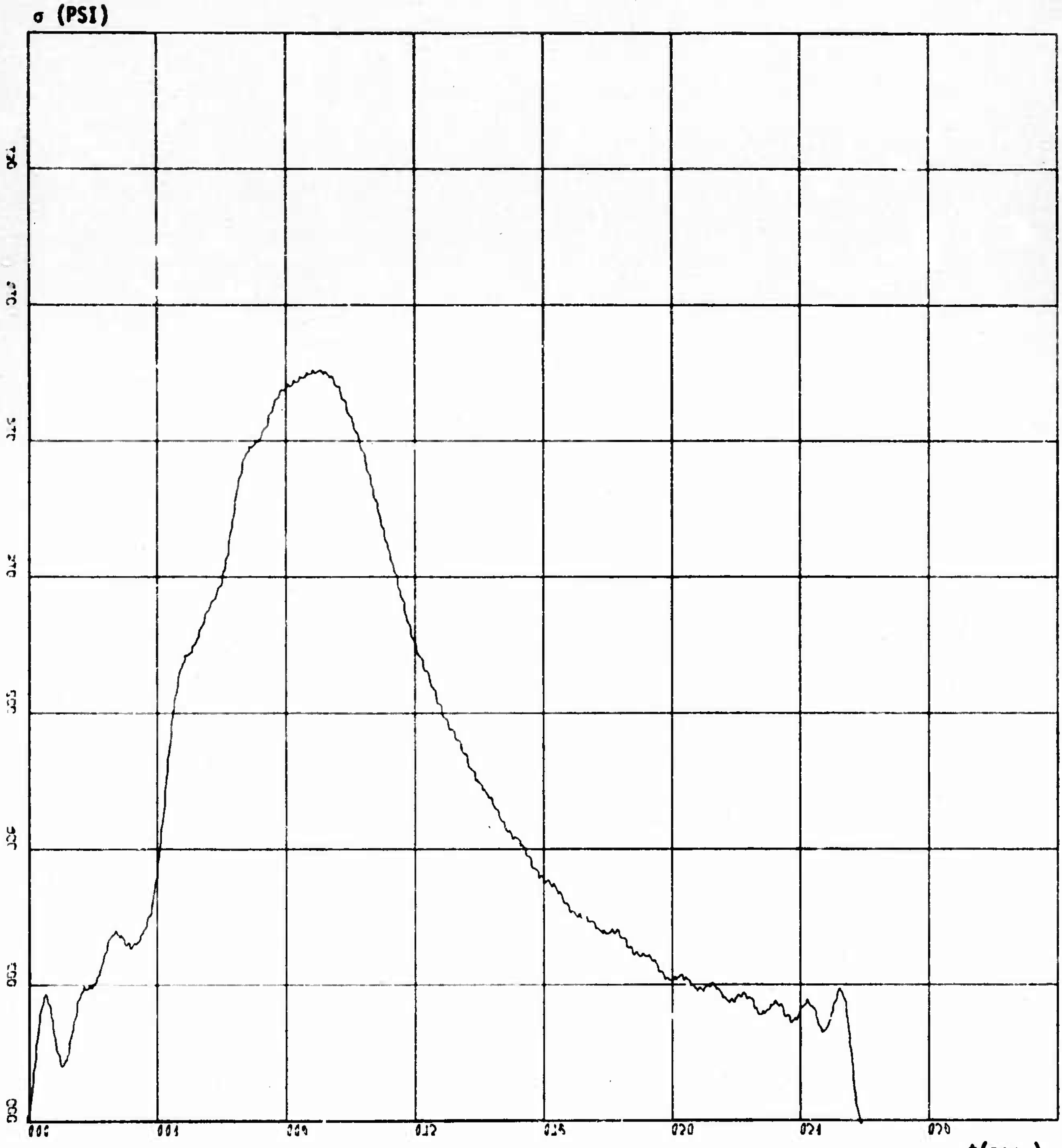



Figure 2

α = .026 sec.

X-SCALE=4.00E-03 UNITS
Y-SCALE=3.00E+01 UNITS
AXIAL WAVE PROPAGATION-STRESSES AT X/L= 0.




α = .026 sec.

Figure 3

X-SCALE=4.00E-03 UNITS
Y-SCALE=3.00E+01 UNITS
AXIAL WAVE PROPAGATION-STRESSES AT X/L= 1/4

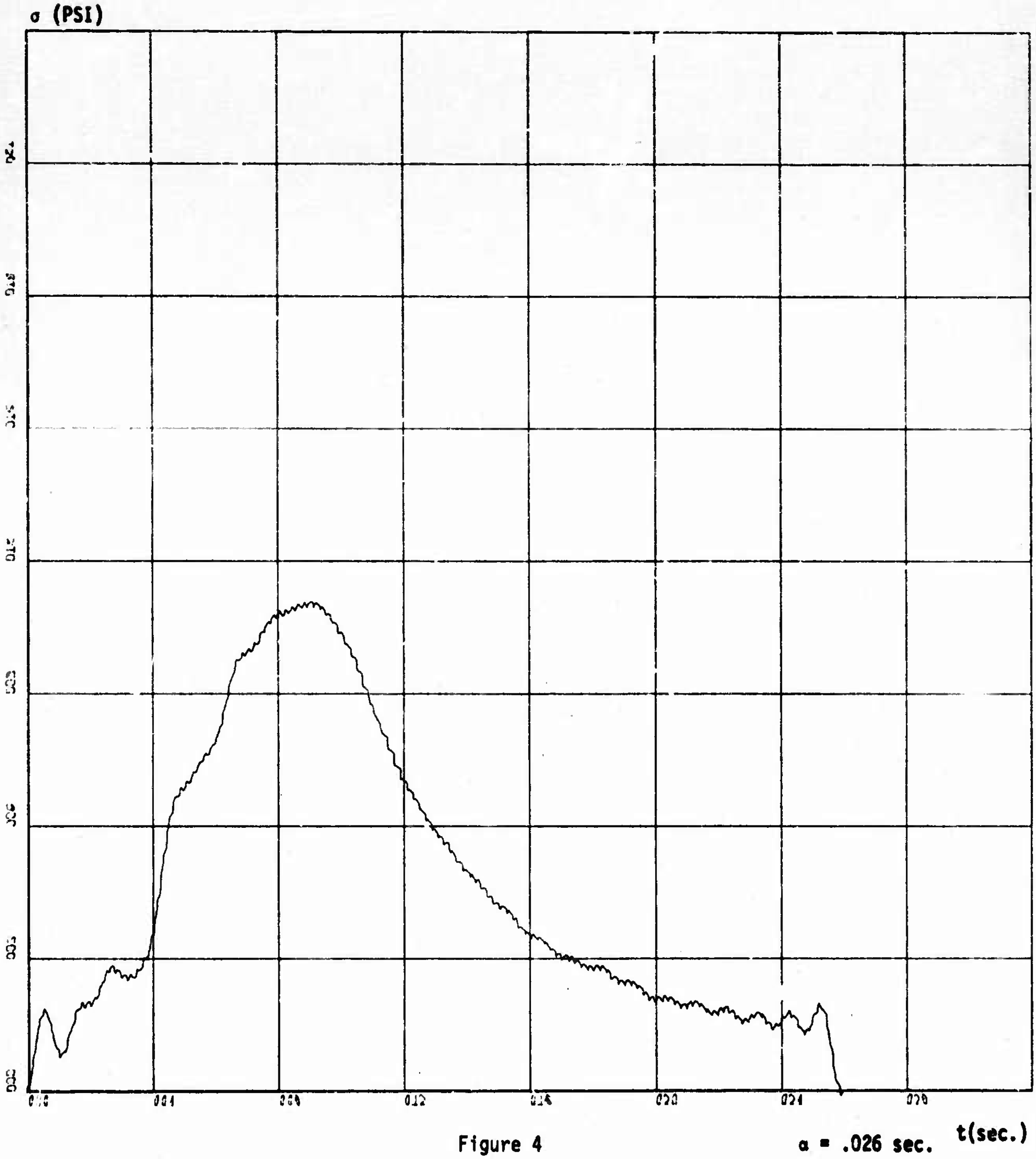
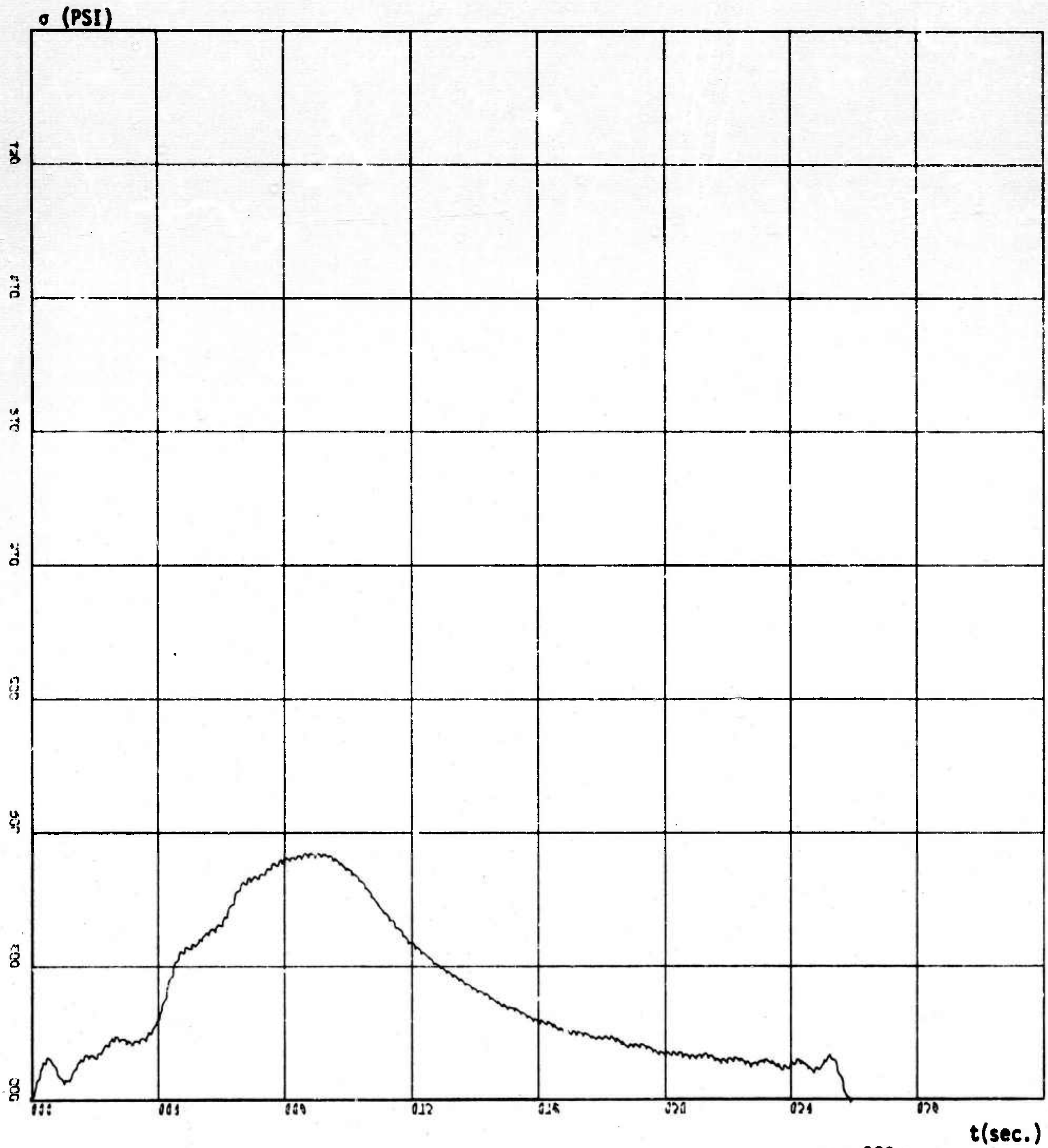


Figure 4

α = .026 sec.

K-SCALE=4.00E-03 UNITS
Y-SCALE=3.00E+01 UNITS
AXIAL WAVE PROPAGATION-STRESSES AT X/L = 1/2




$\alpha$ = .026 sec.

Figure 5

X-SCALE=4.00E-03 UNITS
Y-SCALE=3.00E+01 UNITS
AXIAL WAVE PROPAGATION-STRESSES AT X/L= 3/4

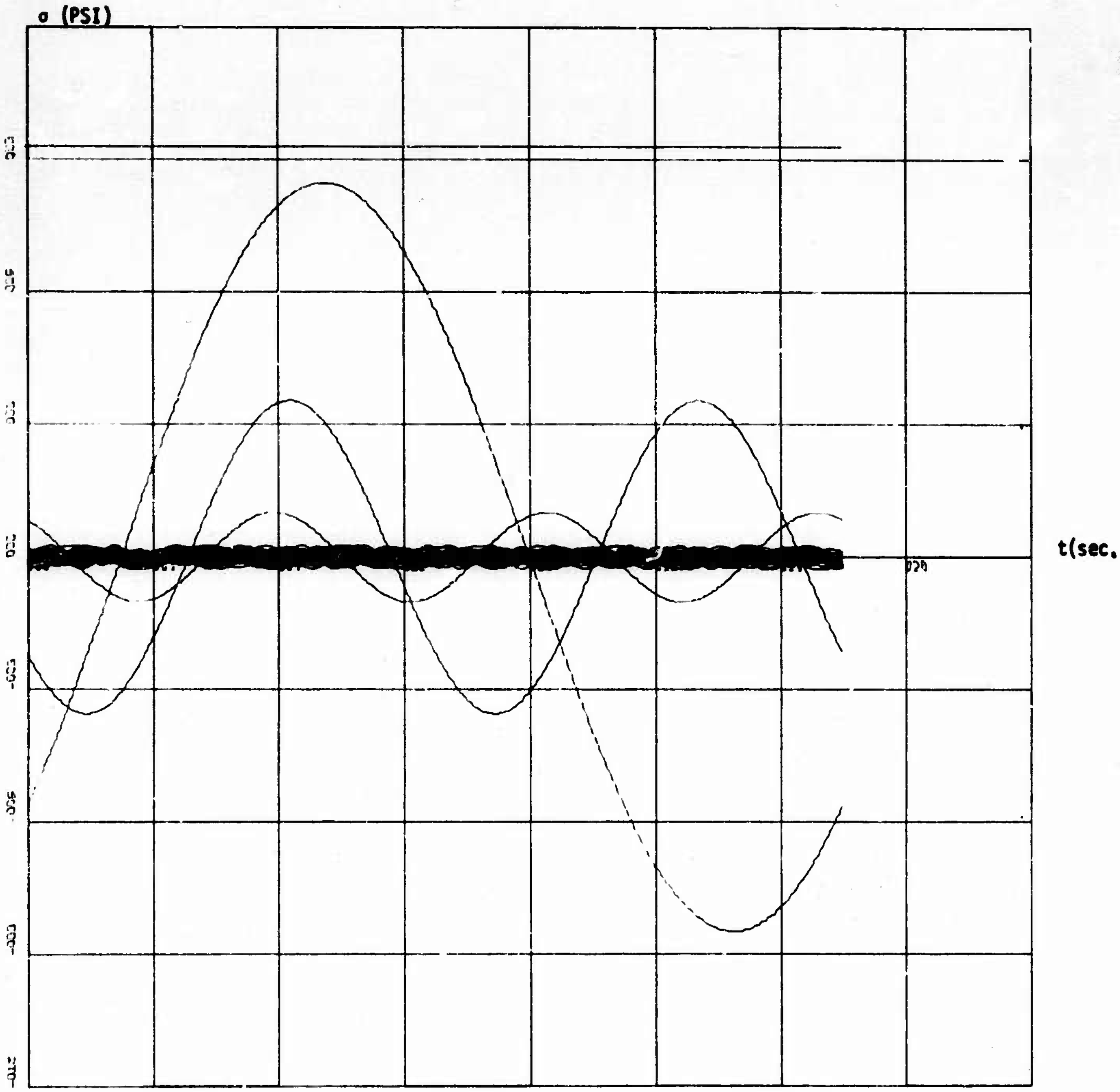
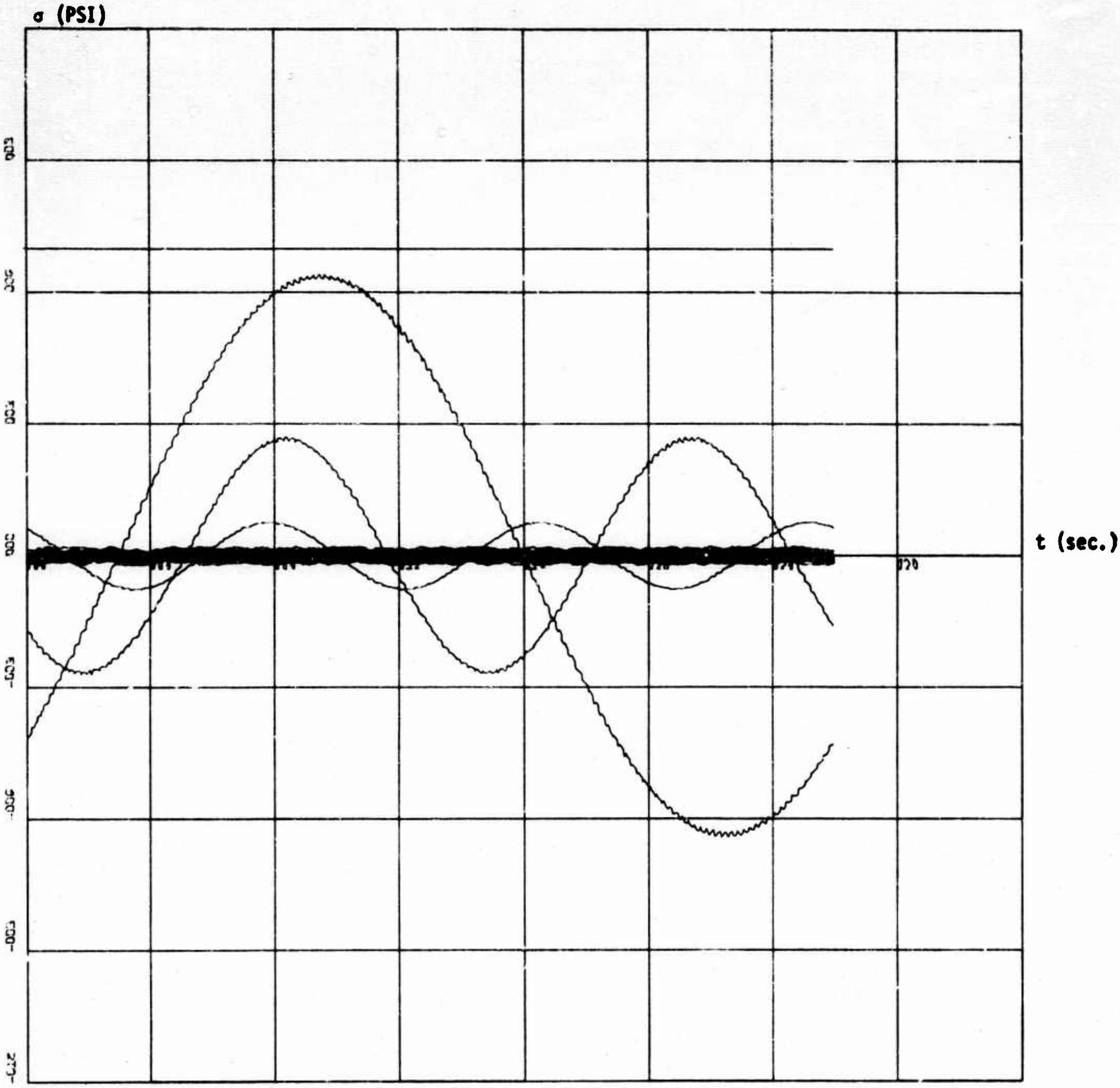



Figure 6

α = .026 sec.

X-SCALE=4.00E-03 UNITS
Y-SCALE=3.00E+01 UNITS
AXIAL WAVE PROPAGATION-HARMONICS at X/L = 0.




α = .026 sec.

Figure 7

X-SCALE=4.00E-03 UNITS
Y-SCALE=3.00E+01 UNITS
AXIAL WAVE PROPAGATION-HARMONICS at X/L = 1/4

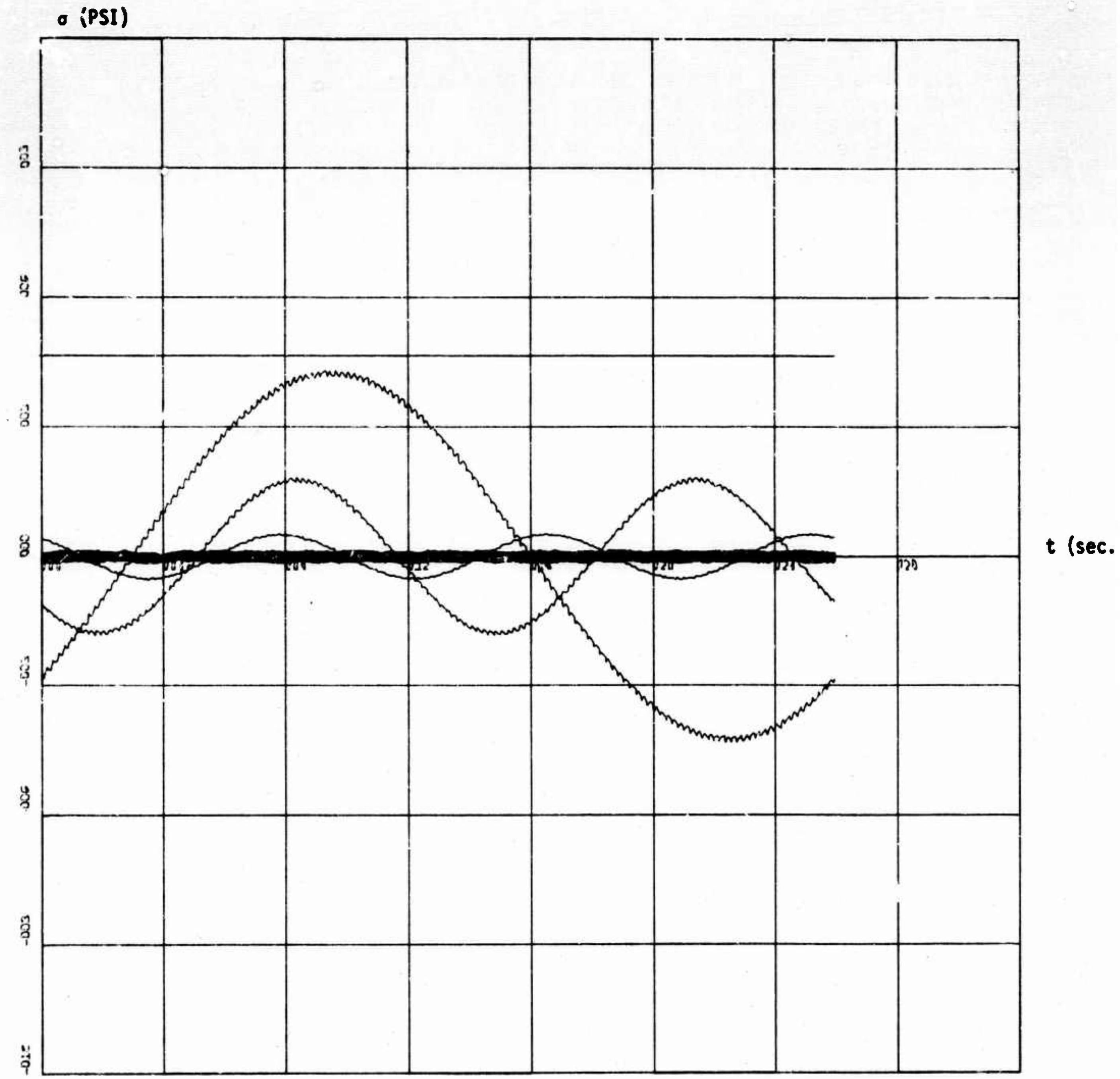
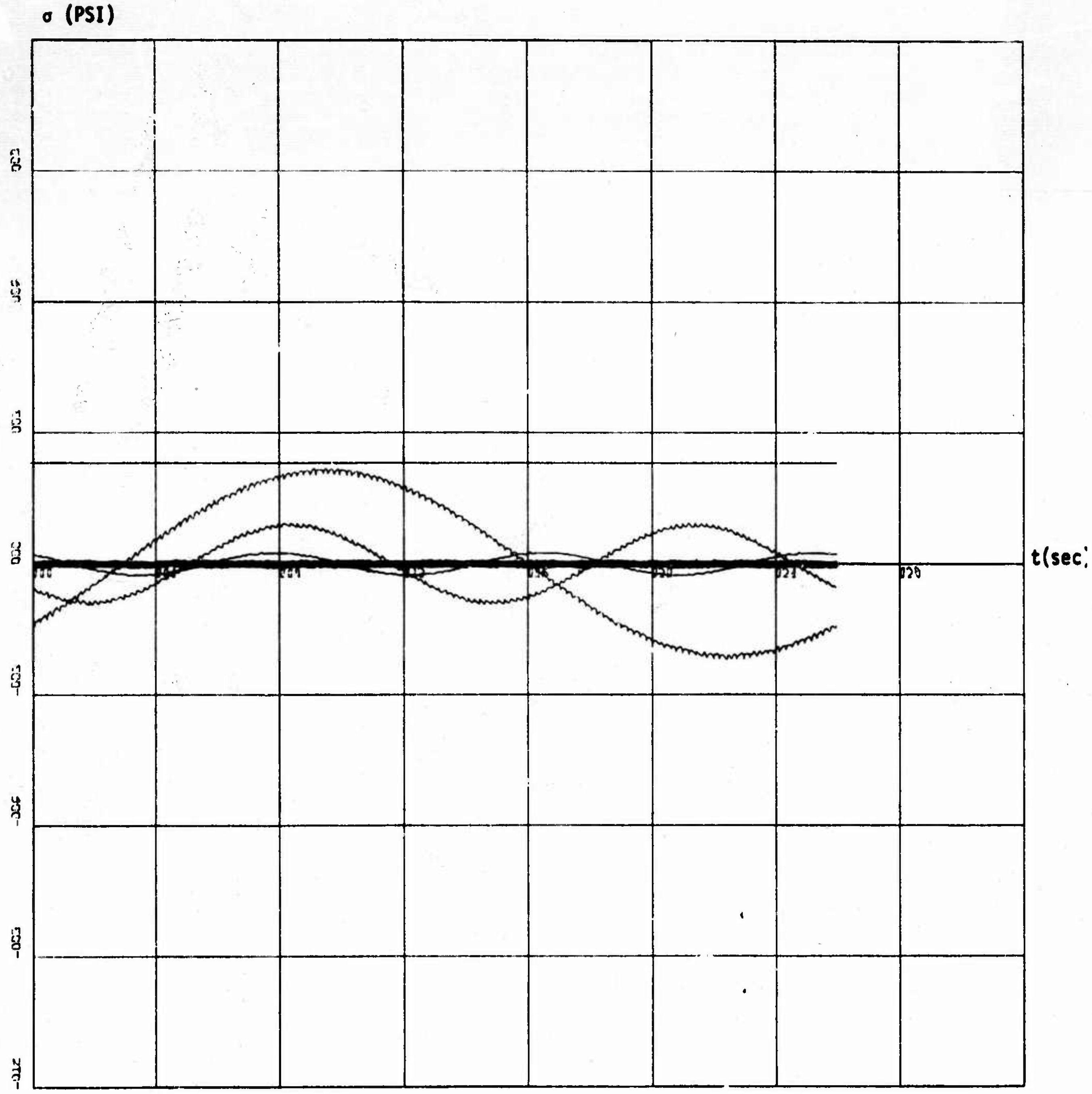



Figure 8 α = .026 sec.

X-SCALE=4.00E-03 UNITS
Y-SCALE=3.00E+01 UNITS
AXIAL WAVE PROPAGATION-HARMONICS at X/L = 1/2




α = .026 sec.

Figure 9

X-SCALE=4.00E-03 UNITS
Y-SCALE=3.00E+01 UNITS
AXIAL WAVE PROPAGATION-HARMONICS at X/L = 3/4

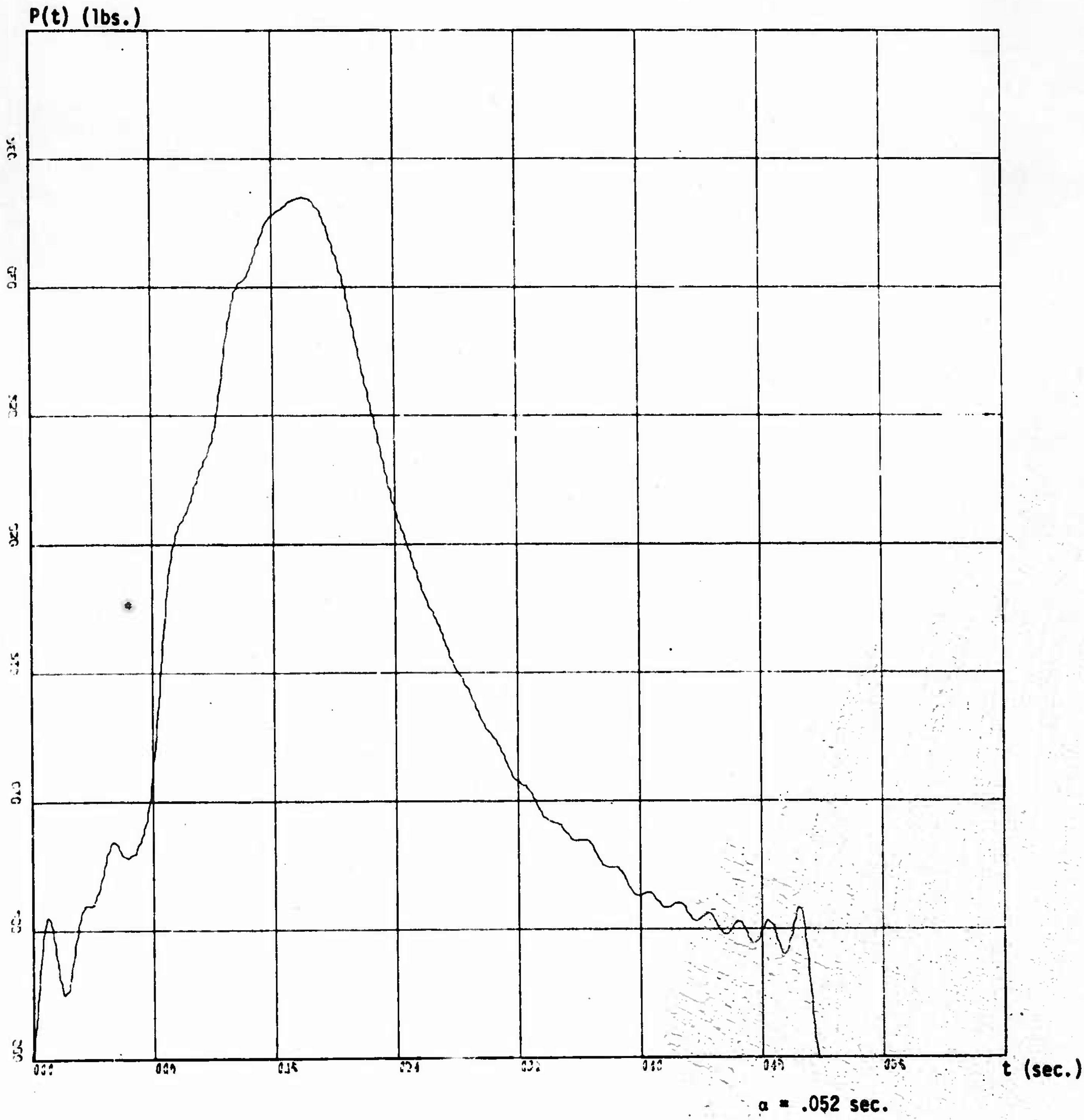



Figure 10

X-SCALE=8.00E-04 UNITS
Y-SCALE=5.00E+00 UNITS
AXIAL WAVE PROPAGATION-BREECH FORCE

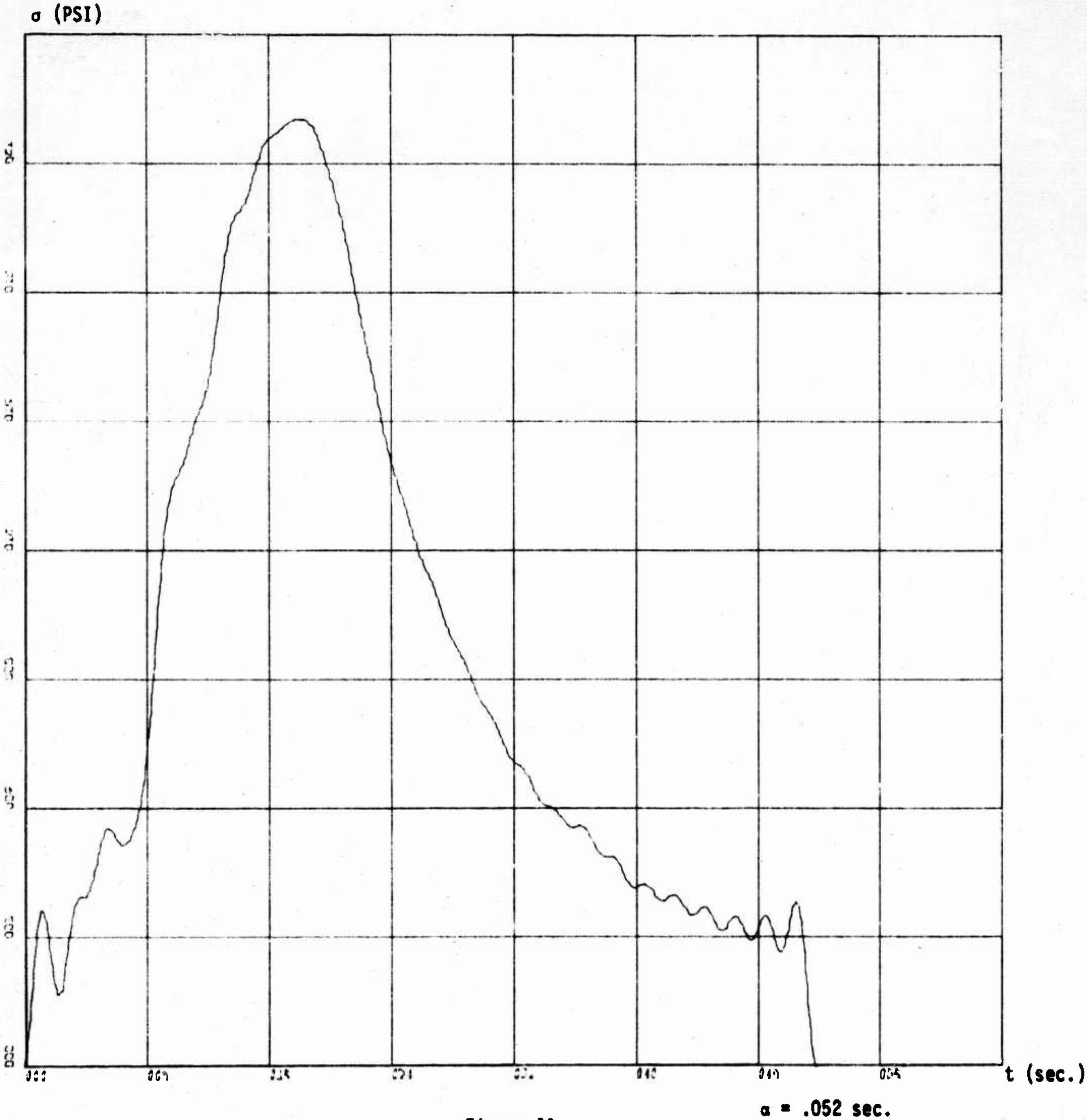



Figure 11

X-SCALE=8.00E-04 UNITS
Y-SCALE=3.00E+01 UNITS

AXIAL WAVE PROPAGATION-STRESSES AT X/L = 0.

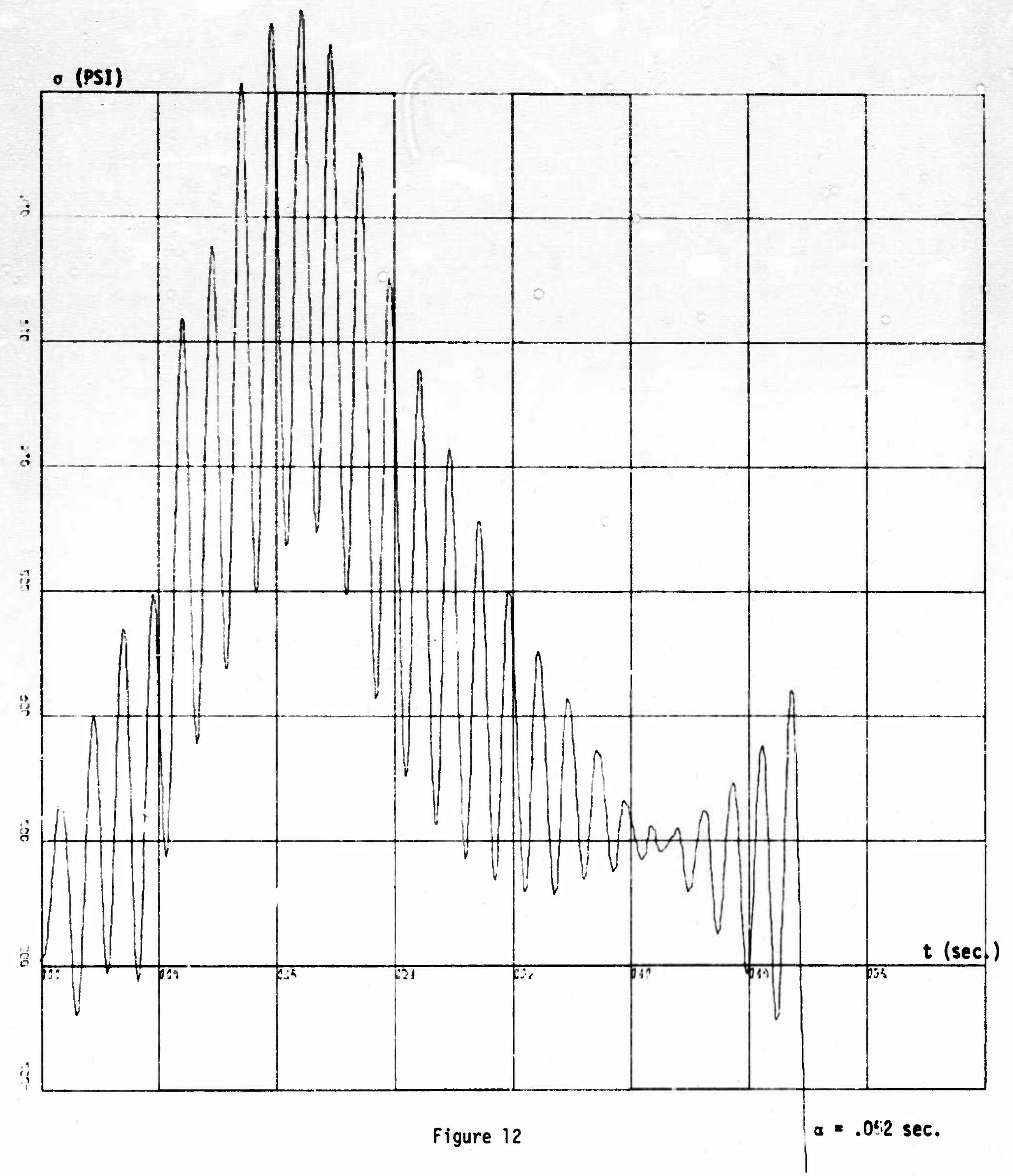



Figure 12

K-SCALE=8.00E-04 UNITS
Y-SCALE=3.00E+01 UNITS
AXIAL WAVE PROPAGATION-STRESSES AT X/L - 1/4.

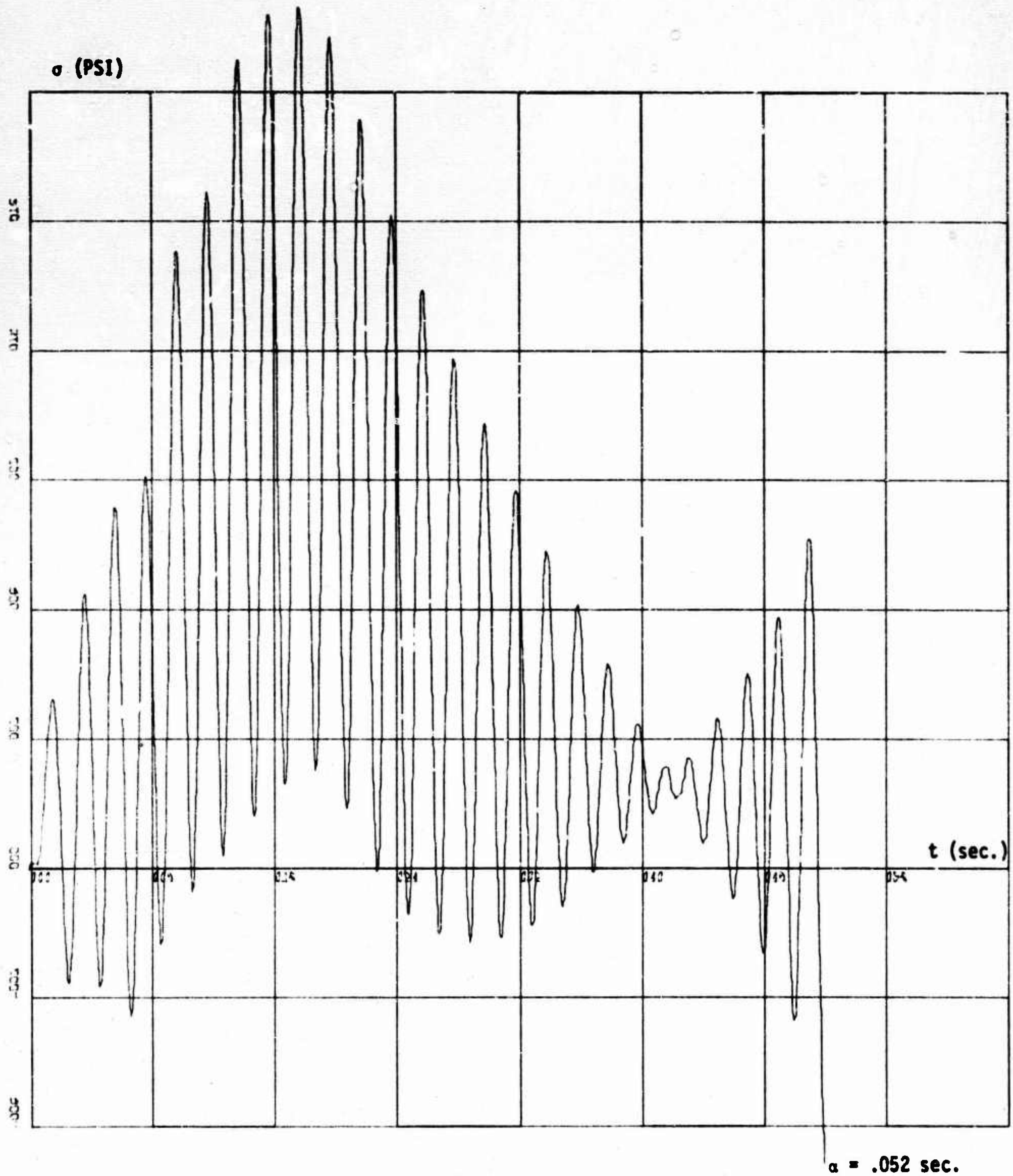
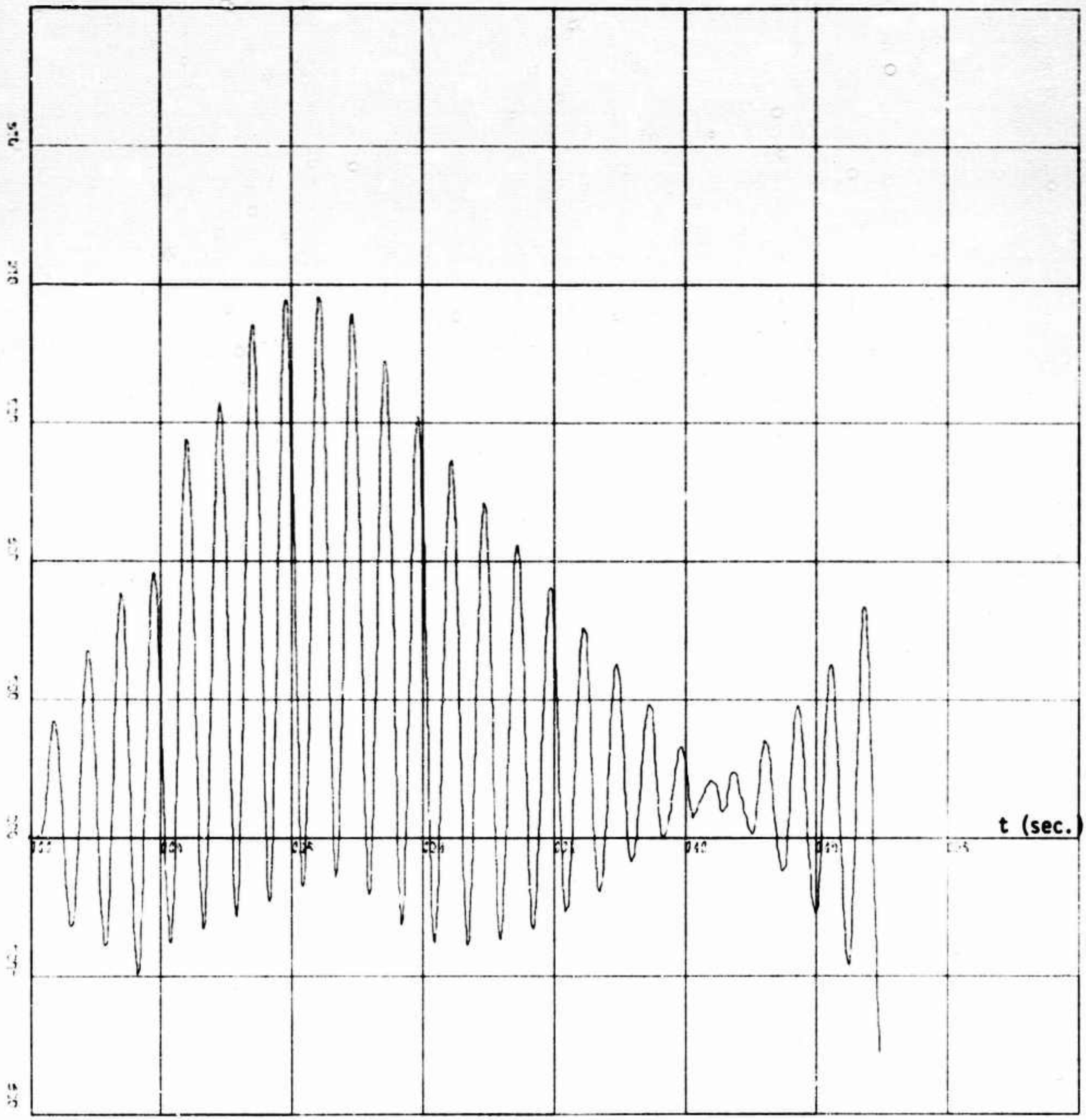



Figure 13

X-SCALE=8.00E-04 UNITS
Y-SCALE=3.00E+01 UNITS
AXIAL WAVE PROPAGATION-STRESSES AT X/L = 1/2.

σ (PSI)




α = .052 sec.

Figure 14

X-SCALE-8.00E-04 UNITS
Y-SCALE-3.00E+01 UNITS
AXIAL WAVE PROPAGATION-STRESSES AT X/L = 3/4.

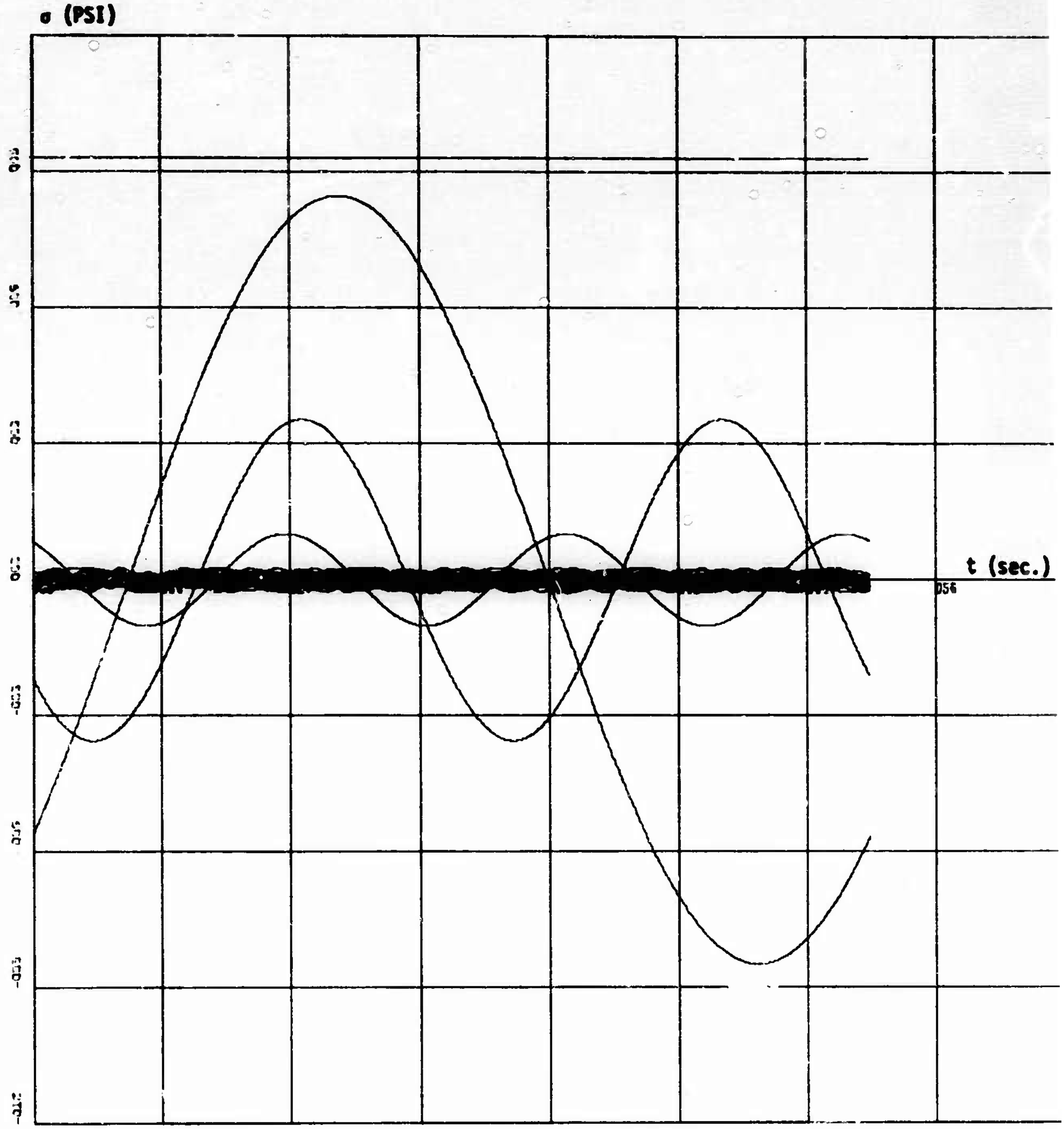



Figure 15

α = .052 sec.

X-SCALE=8.00E-04 UNITS
Y-SCALE=3.00E+01 UNITS
AXIAL WAVE PROPAGATION-HARMONICS at X/L = 0.

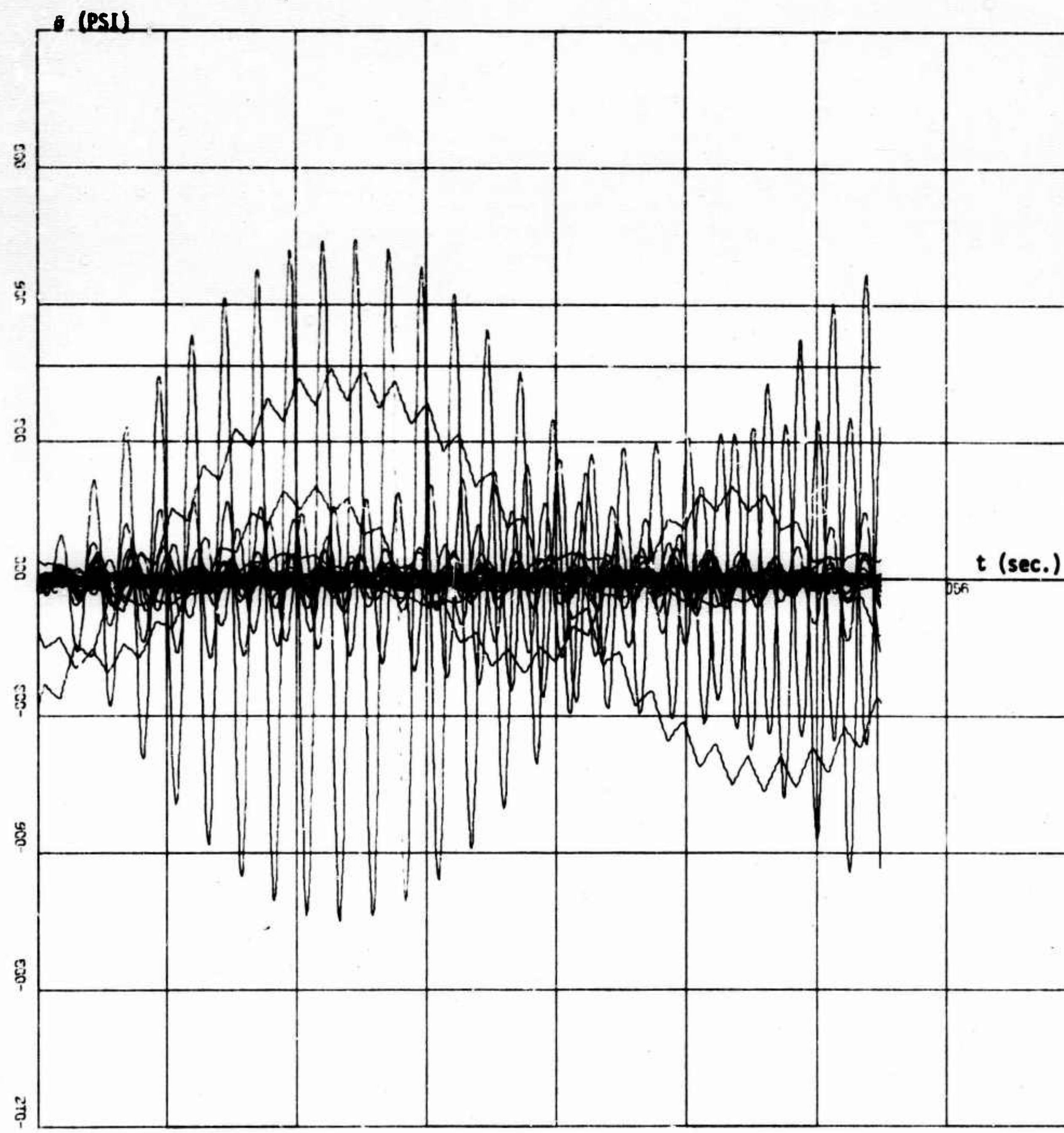



Figure 16

α = .052 sec.

X-SCALE=8.00E-04 UNITS
Y-SCALE=3.00E+01 UNITS
AXIAL WAVE PROPAGATION-HARMONICS at X/L = 1/2.